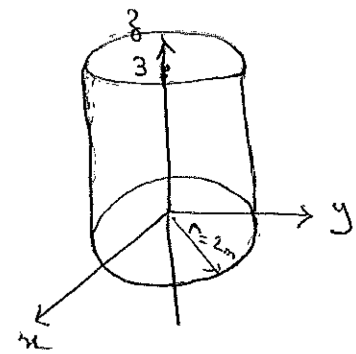
HW6

Problem 4.2

$0 \leq r \leq 2 \qquad \rho_v = 20rz \times 10^{-3}\ C/m^3$

$0 \leq z \leq 3$



$$Q = \int_v \rho_v \, dv$$

$$dv = r\,dr\,d\varphi\,dz \qquad r\Big|_0^2 ,\ \phi\Big|_0^{2\pi} ,\ z\Big|_0^3$$

$$Q = \int_0^3 \int_0^{2\pi} \int_0^2 20rz \times 10^{-3}\, r\,dr\,d\varphi\,dz$$

$$Q = 2\times 10^{-2} \int_0^3 \int_0^{2\pi} \int_0^2 r^2 z\,dr\,d\varphi\,dz = 4\pi \times 10^{-2}\ \frac{r^3}{3}\Big|_0^2\ \frac{z^2}{2}\Big|_0^3$$

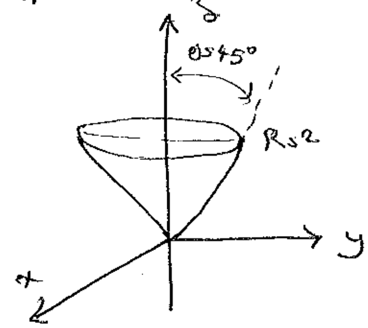
$$= \frac{2\pi}{300}(8)(9) = \frac{48\pi}{100} \text{ Coulombs}$$

Problem 4.3

$0 \leq R \leq 2$

$0 \leq \theta \leq \frac{\pi}{4}$

$\rho_v = 10R^2 \cos^2\theta \times 10^{-3} \; C/m^3$



$$Q = \int_v \rho_v \, dv$$

$$dv = R^2 \sin\theta \, dR \, d\theta \, d\varphi$$

$$R\Big|_0^2, \quad \theta\Big|_0^{\frac{\pi}{4}}, \quad \phi\Big|_0^{2\pi}$$

$$Q = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{2} 0.01 \, R^4 \cos^2\theta \sin\theta \, dR \, d\theta \, d\varphi$$

$$= 0.01 \, \frac{R^5}{5} \Big|_0^2 \, \phi \Big|_0^{2\pi} \int_0^{\frac{\pi}{4}} \cos^2\theta \sin\theta \, d\theta$$

$$= 0.01 \frac{(32)}{5} (2\pi) \left(\frac{\cos^3 0 - \cos^3 \frac{\pi}{4}}{3} \right)$$

$$= \frac{64\pi}{1500} \left[1 - \left(\frac{\sqrt{2}}{2}\right)^3 \right]$$

$$= 86.5 \text{ m coulomb}$$

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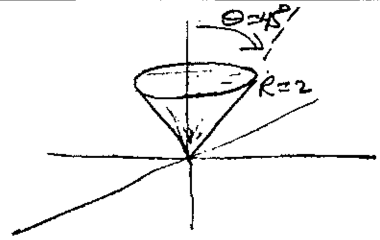
$0 \le R \le 2$
$0 \le \theta \le \pi/4$

$\rho_v = 10R^2 \cos^2\theta \times 10^{-3}\ C/m^3$



$Q_{encl} = \int_v \rho_v \, dv \qquad dv = R^2 \sin\theta \, dR\, d\theta\, d\phi$

$R\Big|_0^2, \ \theta\Big|_0^{\pi/4}, \ \phi\Big|_0^{2\pi}$

$$Q_{encl} = \int_0^{2\pi}\int_0^{\pi/4}\int_0^2 0.01\, R^4 \cos^2\theta \sin\theta \, dR\, d\theta\, d\phi$$

$$= 0.01 \frac{R^5}{5}\Big|_0^2 \phi\Big|_0^{2\pi} \underbrace{\int_0^{\pi/4} \cos^2\theta \sin\theta\, d\theta}_{\longrightarrow\ -\frac{\cos^3\theta}{3}\Big|_0^{\pi/4} =}$$

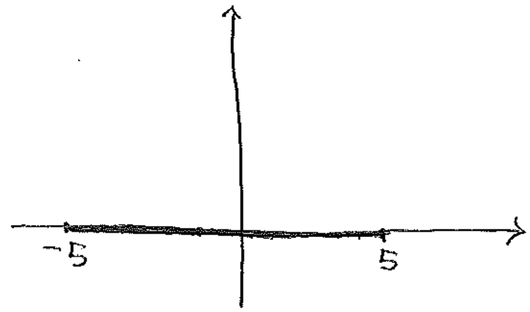
$$= 0.01\left(\frac{32}{5}\right)(2\pi)\left(\frac{\cos^3 0 - \cos^3\frac{\pi}{4}}{3}\right)$$

$$= \frac{64\pi}{1500}\left[1 - \left(\frac{\sqrt{2}}{2}\right)^3\right]$$

$= 86.5\ \text{mCoulomb}$

problem 4.4

$\rho_\ell = 24\, y^2 \times 10^{-3}\ \frac{C}{m}$ $\qquad y\Big|_{-5}^{5}$



$d\ell = dy$

$$Q = \int \rho_\ell \, d\ell$$

$$Q = \int_{-5}^{5} 0.024\, y^2\, dy = \frac{24}{1000} \, \frac{y^3}{3}\Big|_{-5}^{5} = \frac{4(125)-(-125)}{500}$$

$Q = 2$ coulomb

Problem 4.5

circular disk $0 \leqslant r \leqslant a$, $z = 0$

a) $\rho_s = \rho_{s_0} \cos\varphi \ \frac{C}{m^2}$

$$Q = \int_0^{2\pi}\int_0^a \rho_{s_0} \cos\varphi \, r \, dr \, d\varphi = \rho_{s_0} \int_0^{2\pi}\int_0^a r \sin\varphi \, dr \, d\varphi = \rho_{s_0} \frac{r^2}{2}\Big|_0^a \underbrace{\sin\varphi\Big|_0^{2\pi}}_{1-1=0}$$

$= 0$ Coulombs

b) $\rho_s = \rho_{s_0} \sin^2\varphi \ \frac{C}{m^2}$

$$Q = \frac{\rho_{s_0} a^2}{2} \int_0^{2\pi} \sin^2\varphi \, d\varphi = \frac{\rho_{s_0} a^2}{2} \left[\frac{\Phi}{2} - \frac{1}{4} \sin 2\Phi \right]_0^{2\pi}$$

$$= \frac{\rho_{s_0} a^2}{2} \pi$$

c) $\rho_s = \rho_{s_0} e^{-r} \ \frac{C}{m^2}$

$$Q = \rho_{s_0} \int_0^a \int_0^{2\pi} r e^{-r} \, dr = 2\pi \rho_{s_0} \left[e^{-r}(-r-1) \right]_0^a$$

$Q = 2\pi \rho_{s_0} \left[1 - (1+a) e^{-a} \right]$ Coulombs

d) $\rho_s = \rho_{s_0} e^{-r} \sin^2\varphi \ \frac{C}{m^2}$

$$Q = \rho_{s_0} \int_0^{2\pi}\int_0^a r e^{-r} \sin^2\varphi \, dr \, d\varphi = \rho_{s_0} \left[\frac{\Phi}{2} - \frac{1}{4} \sin 2\Phi \right]_0^{2\pi} \left[e^{-r}(1+r) \right]_a^0$$

$= \rho_{s_0} \pi \left[1 - (1+a) e^{-a} \right]$ Coulombs

Problem 4.7

$$J = \frac{5}{R}\,\hat{a}_R \quad \frac{A}{m^2}$$

$$ds = R^2 \sin\theta \, d\theta \, d\varphi \; \hat{a}_R$$

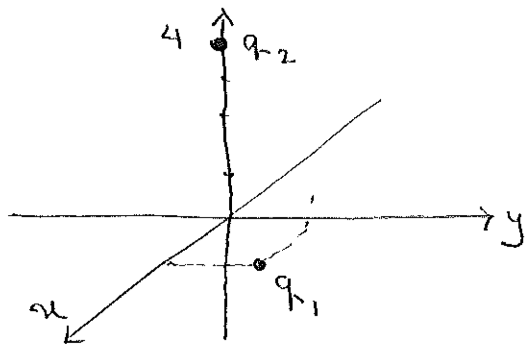
$$I = \int_s \vec{J}\cdot\vec{ds} = \int_0^{\pi}\int_0^{\pi} \frac{5}{R}\,\hat{a}_R \cdot R^2 \sin\theta\,\hat{a}_R \, d\theta\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{\pi} 5R \Big|_{R=5} \sin\theta \, d\theta \, d\varphi$$

$$= 25\,(2\pi)\,(2)$$

$I = 100\pi$ Amperes

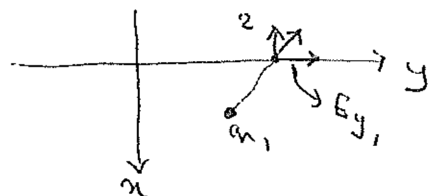
Problem 4.11



$q_1 = 6\mu C$ at $(1_{cm}, 1_{cm}, 0)$

$q_2 = ?$ at $(0, 0, 4\,cm)$

desire $\vec{E}\Big|_{(0, 2cm, 0)}$ to have $E_y = 0$

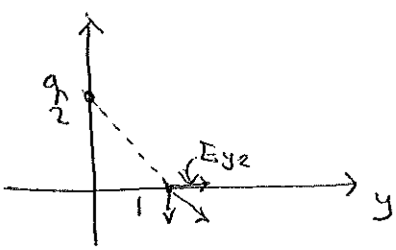


q_1 results in $-\hat{a}_x + a_y$

q_2 results in $+\hat{a}_y - \hat{a}_z$

$R_1 = (-\hat{a}_x + a_y) \times 10^{-2}\,m$

$R_1 = \sqrt{2} \times 10^{-2}\,m$



$R_2 = (2\hat{a}_y - 4\hat{a}_z) \times 10^{-2}\,m$

$R_2 = 2\sqrt{5} \times 10^{-2}\,m$

$\therefore E_{y_1} = -E_{y_2}$ $\therefore q_2$ must be negative (to have a $-\hat{a}_y$ component)

for $q_1 \Rightarrow \vec{E}_1 = \dfrac{q_1}{4\pi\epsilon R_1^2}\hat{a}_{R_1} = \dfrac{q_1 \vec{R}_1}{4\pi\epsilon R_1^3} = \dfrac{q_1}{4\pi\epsilon}\dfrac{(-\hat{a}_x + \hat{a}_y)10^{-2}}{(2)^{3/2}(10^{-2})^3}$

$$E_{y_1} = \frac{6\times10^{-6}(10^4)}{4\pi\epsilon(2)^{3/2}} = \frac{3\times10^{-2}}{2\pi\epsilon\sqrt{8}}$$

for $q_2 \Rightarrow \vec{E}_2 = \dfrac{q_2}{4\pi\epsilon R_2^2}\hat{a}_{R_2} = \dfrac{q_2\vec{R}_2}{4\pi\epsilon R_2^3} = \dfrac{q_2(2\hat{a}_y - 4\hat{a}_z)10^{-2}}{4\pi\epsilon\, 8\sqrt{125}\times10^{-6}}$

$$E_{y_2} = \frac{2(10^4)q_2}{4\pi\epsilon\, 8\sqrt{125}}$$

for $E_{y_1} = -E_{y_2}$

$$\Rightarrow q_2 = -\left(\frac{3\times10^{-2}}{2\pi\epsilon\sqrt{8}}\right)\left(\frac{2\pi\epsilon\, 8\sqrt{125}}{10^4}\right) = -3\sqrt{8}\sqrt{125}\times10^{-6}$$

$q_2 = -30\sqrt{10}\times10^{-6} = -94.88\,\mu$Coulombs